Banach Spaces
Espaces de Banach
(Org: **Robb Fry** (Thompson Rivers) and/et **Srinivasa Swaminathan** (Dalhousie))

RAZVAN ANISCA, Lakehead University, Thunder Bay, Ontario, Canada
Some remarks on ergodic Banach spaces

A separable Banach space X is said to be ergodic if the relation E_0 of eventual agreement between sequences of 0's and 1's is Borel reducible to isomorphism between subspaces of X. This means that there exists a Borel map f mapping elements of 2^ω to subspaces of X such that $\alpha E_0 \beta$ if and only if $f(\alpha) \simeq f(\beta)$. In particular, an ergodic Banach space X must contain 2^ω mutually non-isomorphic subspaces.

We present a constructive version of a recent result of Dilworth, Ferenczi, Kutzarova and Odell regarding the ergodicity of strongly asymptotic ℓ_p spaces.

MAXIM BURKE, University of Prince Edward Island, Department of Mathematics and Statistics, 550 University Avenue, Charlottetown, PE
Kadec norms on spaces of continuous functions

We study the existence of pointwise Kadec renormings for Banach spaces of the form $C(K)$. (A pointwise Kadec renorming is a norm equivalent to the sup norm for which the norm topology and the topology of pointwise convergence agree on the unit sphere.) We show in particular that such a renorming exists when K is any product of compact linearly ordered spaces, extending the result for a single factor due to Haydon, Jayne, Namioka and Rogers. We show that if $C(K_1)$ has a pointwise Kadec renorming and K_2 belongs to the class of spaces obtained by closing the class of compact metrizable spaces under inverse limits of transfinite continuous sequences of retractions, then $C(K_1 \times K_2)$ has a pointwise Kadec renorming. We also prove a version of the three-space property for such renormings.

This is joint work with W. Kubis and S. Todorcevic.

MONICA COJOCARU, University of Guelph
Solvability of Projected Equations in Banach Spaces

We will discuss the solvability of a class of nonlinear differential equations on Banach spaces that relate to variational inequalities and complementarity problems. Such equations have been recently formulated in B-spaces, but their solvability has not yet been discussed. We offer a first insight into the question of existence of solutions for such equations and its implications for the study of applied problems related to such equations. They are a generalization of similar equations in Hilbert spaces, now widely used in applied equilibrium problems in networks, game theoretic and economic problems.

STEPHEN DILWORTH, Department of Mathematics, University of South Carolina, Columbia, SC 29208
Discretization of coefficients in Banach spaces

Let (e_i) be a bounded sequence in a Banach space X such that the additive group G generated by (e_i) is ρ-dense in the unit ball of X, where $\rho < 1$. When (e_i) is a bounded minimal system then ℓ_1 embeds into X^*. If, in addition, the approximant from G may be chosen by a natural algorithm then c_0 embeds into X. We discuss these results and attempt to generalize them to redundant systems such as frames.

This is joint work with Casazza, Odell, Schlumprecht, and Zsák.

NIGEL KALTON, University of Missouri, Columbia, MO 65211, USA
Extension of linear maps into $C(K)$-spaces

We will describe some recent work on the problem of extending linear maps into $C(K)$-spaces. We will also discuss separable Banach spaces X which can only be embedded into a $C(K)$-space (with K compact metric) in one way up to automorphism; these spaces include c_0 and ℓ_1 not ℓ_p when $p > 1$.

LEE KEENER, University of Northern British Columbia, Prince George, BC
Approximation by Lipschitz, analytic functions on certain Banach spaces

In 1934, H. Whitney showed that continuous, real-valued functions defined on open sets can be uniformly approximated by analytic maps. Subsequently (1953), Kurzweil demonstrated that in a separable Banach space admitting a separating polynomial, any continuous function can be uniformly approximated by maps analytic on the space. In the spirit of Kurzweil, we show that in such spaces any uniformly continuous, real-valued function can be uniformly approximated by Lipschitz analytic maps on bounded sets.

EDWARD ODELL, Department of Mathematics, The University of Texas at Austin, 1 University Station C1200, Austin, TX 78712-0257
Systems formed by translations of one element in $L_p(\mathbb{R})$

Let $f \in L_p(\mathbb{R})$ and for $\lambda \in \mathbb{R}$ let $f_{(\lambda)}$ be the translation of f by λ,

$$f_{(\lambda)}(t) = f(t - \lambda).$$

We study the Banach space

$$X(f, \Lambda) = [\{f_{(\lambda)} : \lambda \in \Lambda\}],$$

the closed linear span in $L_p(\mathbb{R})$ of the set $\{f_{(\lambda)} : \lambda \in \Lambda\}$, where $\Lambda \subseteq \mathbb{R}$ is discrete under the assumption that $(f_{(\lambda)})_{\lambda \in \Lambda}$ is basic (under some ordering), unconditional basic, a Schauder frame (under some ordering) or part of a biorthogonal system.

Some results we obtain are

(1) If $(f_{(\lambda)})_{\lambda \in \Lambda}$ is a basis (or Schauder frame) for $X(f, \Lambda) \subseteq L_1(\mathbb{R})$ then $X(f, \Lambda)$ embeds isomorphically into ℓ_1.

(2) If $1 < p \leq 2$ and $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic then it is equivalent to the unit vector basis of ℓ_p.

(3) If $2 < p \leq 4$ and $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic then $X(f, \Lambda)$ embeds into ℓ_p.

(4) For $4 < p < \infty$ there exists $f \in L_p(\mathbb{R})$ and $\Lambda \subseteq \mathbb{N}$ so that $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic and $X(f, \Lambda)$ contains an isomorph of $L_p(\mathbb{R})$.

We report on recent joint work with Th. Schlumprecht, B. Sari and B. Zheng.

ALEXEY POPOV, University of Alberta, Edmonton, Alberta
Finitely strictly singular operators between James spaces

In this talk we will show that the strictly singular operator without invariant subspaces constructed by C. J. Read is finitely strictly singular. This result is obtained from the following fact: if $k \leq n$ then every k-dimensional subspace of $\mathbb{R}^n$ contains a zigzag of order k, that is, a vector $x = (x_i)_{i=1}^n$ with $|x_i| \leq 1$ for all i such that $x_{m_i} = (-1)^i$ for some $m_1 < m_2 < \cdots < m_k$.

THOMAS SCHLUMPRECHT, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368
On the sampling and recovery of bandlimited functions via scattered translates of the Gaussian

Let λ be a positive number, and let $(x_j : j \in \mathbb{Z}) \subset \mathbb{R}$ be a fixed Riesz-basis sequence, namely, (x_j) is strictly increasing, and the set of functions $\{\mathbb{R} \ni t \mapsto e^{ix_j t} : j \in \mathbb{Z}\}$ is a Riesz basis (i.e., unconditional basis) for $L_2(-\pi, \pi)$. Given a function $f \in L_2(\mathbb{R})$ whose Fourier transform is zero almost everywhere outside the interval $[-\pi, \pi]$, there is a unique sequence $(a_j : j \in \mathbb{Z})$ in $\ell_2(\mathbb{Z})$, depending on λ and f, such that the function

$$I_\lambda(f)(x) := \sum_{j \in \mathbb{Z}} a_j e^{-\lambda(x - x_j)^2}, \quad x \in \mathbb{R},$$

is continuous and square integrable on $(-\infty, \infty)$, and satisfies the interpolatory conditions $I_\lambda(f)(x_j) = f(x_j)$, $j \in \mathbb{Z}$. It is shown that $I_\lambda(f)$ converges to f in $L_2(\mathbb{R})$, and also uniformly on $\mathbb{R}$, as $\lambda \to 0^+$.

RICHARD SMITH, Institute of Mathematics of the Academy of Sciences of the Czech Republic
Adequate families and renorming of spaces with unconditional bases

We characterise those spaces X with an (uncountable) unconditional basis which admit an equivalent norm, the dual norm of which is strictly convex. The problem is essentially topological, and the notion of adequate families, introduced by Talagrand, plays a central role. We discuss the corresponding situation for Gâteaux smooth norms and related questions.

This is joint work with S. Troyanski of the Universidad de Murcia in Murcia, Spain.

ADI TCACIUC, University of Alberta, Edmonton, Alberta
Almost invariant half-spaces

We say that a bounded linear operator T on a Banach space X admits an almost invariant half-space if there exists an infinite dimensional and infinite codimensional closed subspace Y (a half-space) and a finite dimensional subspace F such that $T(Y) \subset Y + F$. The question whether every bounded linear operator admits an almost invariant half-space is connected to the invariant subspace problem, but it is not necessarily weaker.

In this talk we introduce a promising technique for approaching this question and prove several positive results for weighted shifts operators. In particular we show that Donoghue operators, which do not have invariant half-spaces, admit almost invariant half-spaces with one dimensional "error".

This is joint work with G. Androulakis, A. Popov and V. Troitsky.

VLADIMIR TROITSKY, University of Alberta
Extensions of Perron–Frobenius Theorem to semigroups of positive operators on Banach lattices

The classical Perron–Frobenius Theorem asserts that if a positive matrix has no (proper non-zero) invariant order ideals (i.e., subspaces spanned by subsets of the standard basis) then the spectral radius of this matrix is an eigenvalue, and the corresponding eigenvector is unique and strictly positive (up to scaling). There has been several important extensions of this result. Instead a positive matrix, one can consider a semigroup of positive operators on a Banach lattice. We prove versions of the Perron–Frobenius Theorem as well as some other interesting properties of such semigroups.