SHIPING LIU, Université de Sherbrooke

On almost projective and almost injective modules

Let A be an artin algebra, and consider the category $\operatorname{mod} A$ of finitely generated right A-modules. A module M in $\operatorname{mod} A$ is called *almost projective* if $\operatorname{Ext}^1_A(M, X) \neq 0$ for at most finitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$; and *almost injective* if $\operatorname{Ext}^1_A(X, M) \neq 0$ for at most finitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$. We shall show that the almost projective or injective modules are distributed in finitely many DTr-orbits in the Auslander–Reiten quiver of A. In particular, A is of finite representation type if and only if every module in $\operatorname{mod} A$ is almost projective or injective. As a consequence, if A is a finite-dimensional algebra over an algebraically closed field of infinite representation type, then there exists infinitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$ which are neither almost projective nor almost injective.