
Model Theory and Applications to Geometry
Théorie des modèles et applications à la géométrie
(Org: **Zoé Chatzidakis** (CNRS) and/et **Patrick Speissegger** (McMaster))

SALIH AZGIN, McMaster University, Hamilton, ON
Model Theory of Valued Difference Fields

We will present some recent developments in the model theory of valued fields equipped with a distinguished automorphism as well as some open problems in the area. This study can be carried out in various interesting contexts which depend on the interaction between the valuation and the distinguished automorphism. We obtain Ax–Kochen type results for “valued difference fields” as well as relative quantifier elimination. Some of the techniques introduced for analyzing these structures turn out to provide some perspective for valued fields of positive characteristic.

LUC BÉLAIR, Université du Québec à Montréal (UQAM), Dép. de mathématiques, C.P. 8888 succ. Centreville, Montréal, Québec, H3C 3P8
Modules de Witt et équations aux différences linéaires

On considère un corps valué (K, v) muni d'une isométrie σ , comme un module valué sur l'anneau de Ore des opérateurs aux différences linéaires en σ . Dans cet exposé, on fixe l'attention sur les séries formelles et les vecteurs de Witt. Avec des hypothèses sur le corps résiduel, on obtient des axiomatisations et l'élimination des quantificateurs dans le langage naturel, ce qui assure qu'on n'a pas la propriété d'indépendance. Dans ce formalisme, on s'arrête aussi sur le transfert entre les vecteurs de Witt et les séries formelles.

En collaboration avec Françoise Point.

FRANCK BENOIST, University of Leeds, Leeds LS2 9JT, UK
Schemes for Hasse rings / Schémas pour les anneaux de Hasse

It is known that in the theory of Closed Hasse Fields, the type-definable sets can be described in a geometric way, *i.e.*, as zero sets of differential polynomials (up to boolean combinations). However, many difficulties arise when we try to give a theory of differential algebraic varieties which relies on schemes. I will explain where these difficulties come from and describe a class of Hasse rings with a nice theory of differential schemes.

On sait que dans la théorie des Corps de Hasse Clos, les ensembles type-définissables peuvent être décrits de manière géométrique, *i.e.*, comme ensembles des zéros de polynômes différentiels (à combinaison booléennes près). Mais de nombreuses difficultés apparaissent quand on essaie de développer une théorie des variétés différentielles algébriques qui s'appuie sur les schémas. J'expliquerai l'origine de ces difficultés et je décrirai une classe d'anneaux de Hasse avec une théorie des schémas différentiels agréable.

ELISABETH BOUSCAREN, Université Paris-Sud 11, Département de Mathématiques d'Orsay, 91405 Orsay, France
Model theory of separably closed fields and semi-abelian varieties

L'étude, du point de vue de la théorie des modèles, d'un corps séparablement clos L de caractéristique $p > 0$ non parfait, amène naturellement à regarder les groupes de la forme $G(L)$, le groupe des points L -rationnels de G , pour G un groupe algébrique défini sur L . Nous parlerons du cas où G est une variété semi-abélienne, et nous présenterons quelques résultats récents concernant le sous-groupe des points infiniment p -divisibles de $G(L)$.

Travail en collaboration avec F. Benoist et A. Pillay.

The study, from the point of view of model theory, of a non-perfect separably closed field L of characteristic $p > 0$, leads naturally to the study of groups of the form $G(L)$, the group of L -rational points of G , for G an algebraic group over L . We will consider the case when G is a semi-abelian variety and present some recent results concerning the subgroup of infinitely p -divisible points of $G(L)$.

Joint work with F. Benoist and A. Pillay.

FRANÇOISE DELON, Université Paris 7, UFR de Mathématiques, 175 rue du Chevaleret, 75013 Paris, France

C-minimal structures

A C -relation is a ternary relation first-order interpreting a tree which is a meet-semi-lattice, the domain of the C -relation being then a covering set of branches with no isolated branch. A set M , equipped with a C -relation and possibly an additional structure, is called C -minimal if any definable subset of M is definable without quantifiers in the pure language of C , and if the same holds in any elementarily equivalent structure.

As an example, a C -relation defined on a field and compatible with the two operations derives from a valuation, in the sense that $C(x, y, z)$ means $v(x - y) < v(y - z)$, and it is C -minimal iff the field is algebraically closed. But a C -relation defined on, and compatible with, a group need not derive from a valuation.

A C -minimal structure is algebraically bounded in the sense that finite uniformly definable sets have a bounded size. On the other hand the algebraic closure need not satisfy the exchange principle. C -minimal structures with exchange are “geometric” in the sense of Zilber. In this context, we may ask the question of the trichotomy: Is it possible to define a group in nontrivial structures? To define a field in nonlocally modular structures? To classify some of these structures?

The aim is to extend once more the range of applications of the powerful machinery of the stability. O -minimality allowed handling some ordered structures and the hope is that C -minimality will do the same with geometric C -minimal structures.

ANDREAS FISCHER, University of Saskatchewan, Dept. of Mathematics & Statistics, 106 Wiggins Road, Saskatoon, Canada S7N 5E6

Definability of algebraic models

By a theorem of Akbulut and King, every smooth compactifiable manifold N (that is, N is diffeomorphic to the interior of a smooth compact manifold with boundary) is diffeomorphic to a non-singular real algebraic set. We say that N admits an algebraic model. We discuss the definability of the diffeomorphism under the assumption that the underlying set of N is definable in an expansion of the real field. In general, every definably compactifiable differentiable (C^k for finite k) manifold admits definably an algebraic model. For structures expanding the real exponential field we obtain stronger results. There, definably compactifiable smooth manifolds admit definably and smooth algebraic models.

ERIC JALIGOT, University of Lyon

Geometry and an abstract theory of Weyl groups

Certains arguments géométriques apparaissent dans des arguments de généricité pour les développements les plus récents et généraux de la théorie abstraite des groupes de Weyl dans les groupes de rang de Morley fini. Cet exposé fera le point sur ces questions.

Some geometric arguments appear in genericity arguments for the most recent and general developments of the abstract theory of Weyl groups in groups of finite Morley rank. This talk will deal with these aspects.

GARETH O. JONES, McMaster University, Hamilton, ON

Model completeness and o-minimality

I shall discuss a model completeness result for polynomially bounded o-minimal expansions of the real field, with some consequences for certain exponential structures. The proof combines Gabrielov's methods with ideas from Wilkie's exponentiation paper.

T. G. KUCERA, Department of Mathematics, University of Manitoba
Model theory for topological modules

In 1975, T. A. McKee first introduced a formal language L_t and associated logic suitable for the study of topological structures; this logic was discovered independently about the same time by S. Garavaglia and by M. Ziegler. They established that L_t has a Compactness Theorem, a Lowenheim–Skolem Theorem, and satisfies a Lindstrom Theorem (amongst other things). Later, in my PhD thesis, I showed how to develop Stability Theory in L_t and developed an analogue in L_t for topological modules to the basic “pp-elimination” of quantifiers result for ordinary modules. The logic L_t is thus basically a generalized first-order logic, and while it captures some topological concepts well, it is too weak to capture the full strength of topology, as most of the important concepts of topology are inherently higher order in nature. Thus L_t has not received a great deal of attention since its initial presentation.

My Masters student Clint Enns and I have been investigating to what extent the well-developed model theory of modules might have a useful analogue in L_t for topological modules. In particular, we ask whether there is a natural and useful concept of pure embedding for topological modules, and if so whether there is a natural concept of pure-injective topological module. We cannot expect a completely parallel development, in part because the logic L_t does not “force” mappings between structures to be continuous, and in part because the topological co-product [direct sum] of topological modules does not embed topologically in the topological product.

SALMA KUHLMANN, University of Saskatchewan, 106 Wiggins Road, Saskatoon, SK S7N 5E6
Exponential-Logarithmic vs. Logarithmic-Exponential series

We explain how the field of logarithmic-exponential series constructed in [DMM1] and [DMM2] embeds as an exponential field in any field of exponential-logarithmic series (constructed in [K] and [KS]). On the other hand, we explain why no field of exponential-logarithmic series embeds in the field of logarithmic-exponential series. This establishes that the two constructions are intrinsically different, in the sense that they produce non-isomorphic models of $T_{an,exp}$.

Joint work with Marcus Tressl.

References

- [DMM1] L. van den Dries, A. Macintyre and D. Marker, *Logarithmic-Exponential power series*. J. London Math. Soc. (2) **56**(1997), 417–434.
- [DMM2] ———, *Logarithmic-Exponential series*. Ann. Pure Appl. Logic **111**(2001), 61–113.
- [K] S. Kuhlmann, *Ordered Exponential Fields*. The Fields Institute Monograph Series **12**, AMS Publications, 2000.
- [KS] S. Kuhlmann and S. Shelah, *κ -bounded Exponential Logarithmic Power Series Fields*. Ann. Pure Appl. Logic **136**(2005), 284–296.

OLIVIER LE GAL, Université de Rennes 1
Un contre exemple à la décomposition cellulaire lisse

La décomposition cellulaire est une des propriétés élémentaires de la géométrie modérée: tout ensemble définissable dans une structure o-minimale admet, pour tout entier k , une décomposition en cellules de classe de différentiabilité k . Cependant, les

exemples de structure o-minimale classiques vérifient tous une propriété de décomposition cellulaire C^∞ . Dans cet exposé, nous présenterons un travail commun avec J.-P. Rolin: la construction d'une structure o-minimale ne vérifiant pas la propriété de décomposition cellulaire lisse.

FRANÇOIS LOESER, Ecole normale supérieure, 45 rue d'Ulm, 75005 Paris
Non Archimedean Geometry and Model Theory

We shall present work in progress in collaboration with E. Hrushovski on the geometry of spaces of stably dominated types in connection with non-archimedean geometry à la Berkovich.

AMADOR MARTIN-PIZARRO, Lyon

MICKAEL MATUSINSKI, Saskatchewan

RAHIM MOOSA, University of Waterloo, Waterloo, Ontario
A non-Kähler essentially saturated complex surface

A compact complex manifold M is viewed as a model-theoretic structure in the language where there is a predicate for each analytic subset of M^n . The manifold is *essentially saturated* if it admits a countable sub-language from which all complex-analytic subsets are definable (with parameters). All compact Kähler manifolds (and their holomorphic images, the *Kähler-type* spaces) are essentially saturated. I will describe some recent joint work with Ruxandra Moraru and Matei Toma in which we show that the converse is not true. We show that Inoue surfaces of type S_M are essentially saturated (though not of Kähler-type).

ALEXANDRE RAMBAUD, Université Paris 7, UFR de Mathématiques, 175 rue du Chevaleret, 75013 Paris, France
Quantifier elimination in some non-quasi-analytic classes

Let \mathcal{F} be a class of real functions, continuous on a compact box B of \mathbb{R}^n and C^n on a finite "regular" partition P_n of the interior of B for all $n \in \mathbb{N}$; let us also suppose that \mathcal{F} is closed by sums, products, compositions, derivations and, in a certain way, by implicit functions.

If \mathcal{F} satisfies a condition of non-degeneration (equivalent to quasi-analyticity in the case of quasi-analytic functions), expressed via model theory, we prove that the complete theory of \mathbb{R} equipped with \mathcal{F} admits quantifier elimination and so is o-minimal.

As a consequence, we will give an example of an o-minimal structure on \mathbb{R} which doesn't admit a C^∞ stratification. (This last result was obtained independently by O. Le Gal and J.-P. Rolin via a geometrical proof.)