Non-Commutative Geometry and K-Theory for Operator Algebras
Géométrie non commutative et K-théorie pour algèbres d'opérateurs
(Org: **Alain Connes** (Collège de France; IHES), **George Elliott** (Toronto) and/et **Andrew Toms** (York))

TEODOR BANICA, Toulouse 3 University
Liberation of complex reflection groups

I will present some recent advances on the liberation problem for complex reflection groups, and its relation with some similar questions from Jones' subfactor theory and Voiculescu's free probability theory.

This is based on joint work with S. Belinschi, J. Bichon, M. Capitaine and B. Collins.

JULIEN BICHON, Université Blaise Pascal, Clermont-Ferrand II, campus des Cézeaux, 63177 Aubière Cedex, FRANCE
Qunatum groups acting on 4 points

Wang's quantum permutation group Q_n is the largest compact quantum group acting on n points. Surprisingly, Q_n is infinite-dimensional if $n > 3$. We will first review the construction of Q_n and some of its basic properties. Then we will report on the classification of quantum subgroups of Q_4, i.e., the classification of compact quantum groups acting on 4 points. The quantum subgroups of Q_4 are subject to a McKay type correspondence, described at the level of algebraic invariants, i.e., the multiplicities of the trivial representation in the tensor powers of the fundamental representation.

This is joint work with T. Banica.

ETIENNE BLANCHARD, Institut de Mathématiques de Jussieu, 175, rue du Chevaleret, F-75013 Paris
Extension of C^*-bundles

We investigate which amalgamated products of continuous C^*-bundles are continuous C^*-bundles and we analyse the involved extension problems for continuous C^*-bundles.

PAULO CARRILLO, University of Paris 7, 175, rue de Chevaleret, Paris, France
Compactly supported analytic indices for Lie groupoids and applications

For any Lie groupoid, I will explain how to construct an analytic index morphism taking values in a modified K-theory group which involves the convolution algebra of compactly supported smooth functions. The construction is performed by using a suitable deformation algebra of smooth functions over the tangent groupoid. This allows in particular to prove a more primitive version of the Connes–Skandalis Longitudinal index Theorem for foliations, that is, an index theorem taking values in a group that can still paired with Cyclic cocycles. As another application, for D a gr-PDO elliptic operator with associated index $\mathrm{ind}\, D \in K_0\big(\mathrm{ci}_c(\mathrm{gr})\big)$, we have that the pairing

$$\langle \mathrm{ind}\, D, \tau \rangle,$$

with τ a bounded continuous cyclic cocycle, only depends on the principal symbol class $[\sigma(D)] \in K^0(A^*\,\mathrm{gr})$. The result is completely general for Étale groupoids. I will discuss some potential applications to the Novikov's conjecture.

BENOIT COLLINS, University of Ottawa & CNRS
On the spectral measure of the sum of elements in a finite von Neumann algebra

Given two self-adjoint $n \times n$ matrices A and B with prescribed eigenvalues, the set of all possible spectral distributions for $A + B$ has been conjectured by Horn and proved by Knutson, Tao, Klyachko and Totaro.

We address the same question when A and B have prescribed spectral measures but lie in an arbitrary II_1 factor, and we give elements of answers in terms of inequalities between the spectral measures.

CLAIRE DEBORD, Université Blaise Pascal–Clermont-Ferrand
Index theory on pseudomanifolds

We will see how one can define a good notion of tangent space for a general stratified pseudomanifold X. The tangent space will no longer be a vector bundle but a groupoid whose C^*-algebra is Poincaré dual to the algebra $C(X)$ of continuous functions on X. I will introduce a variant proof of Atiyah–Singer index theorem using groupoids and we will see that all the ingredients (analytical index, Thom isomorphism and topological index) as well as the proof of the index theorem generalize easily to the case of pseudomanifolds with isolated conical singularities as soon as one uses our notion of tangent space.

GEORGE ELLIOTT, University of Toronto, Department of Mathematics, Toronto, Ontario, M5S 2E4
A universal description of the AF envelope of an irrational rotation C^*-algebra

A simple description is given of an embedding of the irrational rotation C^*-algebra into an AF algebra, in which the AF algebra is the same as that considered by Pimsner and Voiculescu in their pioneering work almost thirty years ago, but the construction avoids the technical complexities of the Pimsner–Voiculescu construction. While the description of the embedding is very simple, and it is in fact determined uniquely by a very simple universal property, the proof that the larger algebra is an AF algebra is rather complicated. Indeed, the proof has so far only been carried out in the generic case.

This is joint work with Zhuang Niu.

HEATH EMERSON, University of Victoria
Lefschetz-type invariants and geometric equivariant KK-theory

We summarize recent work of the speaker and Ralf Meyer. This work aims to develop a theory of higher-dimensional Lefschetz fixed-point theory for geometric morphisms (or correspondences) from a space (a manifold) to itself. Classical fixed point theory studies the intersection of the diagonal X in $X \times X$ with an n-dimensional submanifold (say, the graph of a function from X to X). If these submanifolds are transverse, then the intersection is just a discrete set of points, *i.e.*, a zero-dimensional submanifold of X. More generally we can study the intersection of the diagonal with higher-dimensional submanifolds.

If a dimension $k > n$ submanifold W of $X \times X$ is transverse to the diagonal then it has a "fixed-point set" which is a $k - n$-dimensional (typically disconnected) submanifold of X; if W is oriented in K-theory then so is the fixed-submanifold, so it determines a K-homology class, its Lefschetz invariant. We will relate this Lefschetz invariant to a global invariant of the induced map on K-theory, and the (standard) ring structure on K-theory (which itself comes from the inclusion of the diagonal X in $X \times X$. We will work throughout in equivariant KK-theory, which gives noncommutative results.

THIERRY GIORDANO, Université d'Ottawa, Ottawa
Topological orbit equivalence of free, minimal actions of $\mathbb{Z}^d$ on the Cantor set

In 1959, H. Dye introduced the notion of orbit equivalence and proved that any two ergodic finite measure-preserving transformations on a Lebesgue space are orbit equivalent. He also conjectured that an arbitrary action of a discrete amenable group is orbit equivalent to a $\mathbb{Z}$-action. This conjecture was proved by Ornstein and Weiss and its most general case by Connes, Feldman and Weiss by establishing that an amenable non-singular countable equivalence relation R can be generated by a single transformation, or equivalently is hyperfinite, i.e., R is up to a null set, a countable increasing union of finite equivalence relations.

In the Borel case, Weiss proved that actions of $\mathbb{Z}^d$ are (orbit equivalent to) hyperfinite Borel equivalence relations, whose classification was obtained by Dougherty, Jackson and Kechris. In 1995, Giordano, Putnam and Skau proved that minimal $\mathbb{Z}$-actions on the Cantor set were orbit equivalent to approximately finite (AF) relations and their classification was given.

In this talk I will indicate the main steps of the proof of the general result obtained in a joint effort with H. Matui, I. Putnam and C. Skau and whose statement is the following:

Theorem *Any minimal, free $\mathbb{Z}^d$-action on the Cantor set is affable (i.e., orbit equivalent to AF-relations).*

MARIA-PAULA GOMEZ-APARICIO, IMJ, Université de Paris 7, Equipe Algèbres d'Opérateurs, 175 rue du Chevaleret, 75013 Paris

Twisting property (T) and the Baum–Connes morphism by a non-unitary representation

Let G be a locally compact group and ρ a non-unitary finite dimensional representation of G. We consider tensor products of rho by some unitary representations of G in order to define two Banach group algebras analogous to the group C^*-algebras, $C^*(G)$ and $C^*_r(G)$. We then define a twisting of property (T) in terms of such algebras and we use these property to show that, for most of the Lie groups having property (T), any finite dimensional irreducible non-unitary representation ρ of G is isolated among representations of the form $\rho \otimes \pi$, for π an unitary representation of G. We then calculate the K-theory of such group algebras for a large class of groups satisfying the Baum–Connes conjecture and we show that they behave in the same way as the C^*-algebras, $C^*(G)$ and $C^*_r(G)$, at the level of K-theory.

BENOIT JACOB, Université Paris 6, Institut de Mathématiques de Jussieu, 175 rue du Chevaleret, F-75013 Paris, France

Expressing the distance of unitary orbits of positive elements in terms of the Cuntz semigroup

In (arXiv:0707.2220), A. Ciuperca and G. Elliott have studied, for positive elements of a C^*-algebra of stable rank one, the relation between the usual distance (call it d) of unitary orbits and a notion of "Weyl distance" (call it d_W) expressed in terms of the Cuntz semigroup. Specifically, they established the double inequality

$$\frac{1}{8}d \leqslant d_W \leqslant d.$$

This talk will focus on establishing the equality $d = d_W$ for a certain class of ASH algebras of stable rank one, including the Jiang–Su algebra and AT algebras.

MASOUD KHALKHALI, University of Western Ontario

Hopf cyclic cohomology in braided monoidal categories

This is a report on ongoing work which aims at extending some of the homological tools in noncommutative geometry to braided monoidal categories.

Joint work with A. Pourkia.

RAPHAEL PONGE, University of Toronto

Noncommutative geometry and lower dimensional volumes in Riemannian geometry

In this talk I will explain how we can define the "lower dimensional" volumes of any compact Riemannian manifold as the integrals of local Riemannian invariants. For instance a sense can be given to the area and the length of such a manifold in any dimension. The reasoning is motivated by an idea of Connes and involves in an essential way noncommutative geometry and the analysis of Dirac operators on spin manifolds. However, the ultimate definitions of the lower dimensional volumes don't involve noncommutative geometry or spin structures at all.

IAN PUTNAM, University of Victoria
Projection method tilings, their C^*-algebras and KK-theory

We begin with a short introduction to aperiodic tilings, particularly those obtained by the projection method. We will described the C^*-algebras associated to such systems, as introduced by Jean Bellissard and studied by Forrest, Hunton and Kellendonk. This leads to considering actions of free abelian groups on totally disconnected spaces. In addition, we show the existence of very natural elements of the KK-groups of a pair of such systems. These yield new methods for the computation of their K-theory.

LEONEL ROBERT, Fields Institute, 222 College St., Toronto, ON, Canada
The Cuntz semigroup of 2-dimensional spaces

I will give a description of the Cuntz semigroup of all 2-dimensional spaces. I will then discuss some applications of this computation.

ANDREW TOMS, York University, 4700 Keele St., Toronto, ON
The Cuntz semigroup of a minimal diffeomorphism C^*-algebra

The Cuntz semigroup of a stably finite C^*-algebra is an extension of the Murray–von Neumann semigroup to the realm of positive elements. Alternatively, for C^*-algebras of stable rank one, it is the semigroup, under direct sums, of isomorphism classes of countably generated Hilbert modules over a given C^*-algebra. This semigroup is a crucial invariant for simple separable amenable C^*-algebras, whence a basic interest in its structure.

In this talk I will describe the Cuntz semigroup of the C^*-algebra associated to a minimal diffeomorphism of a smooth compact manifold. Some consequences of this description include a classification of countably generated Hilbert modules over such algebras in terms of K-theory and traces, a similar classification of the closures of unitary orbits of self-adjoints, the confirmation of a conjecture of Blackadar and Handelman concerning dimension functions, and a norm-separable analogue of McDuff's uncountable family of non-isomorphic II_1 factors.