Set Theory and its Applications
Théorie des ensembles et ses applications
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Simultaneous approximation and interpolation of increasing functions by increasing entire functions

When A and B are countable dense subsets of $\mathbb{R}$, it follows from a well-known result of Cantor that $f[A] = B$ for some order-isomorphism f of $\mathbb{R}$. A theorem of K. F. Barth and W. J. Schneider states that f can be taken to be the restriction to $\mathbb{R}$ of an entire function. S. Shelah established a consistent analog of Cantor's result for sets of cardinality $\aleph_1$ by building a model where $2^{\aleph_0} > \aleph_1$ and second category sets of cardinality $\aleph_1$ exist while any two sets of cardinality $\aleph_1$ which are nonmeager in every interval are order-isomorphic. In earlier work, we proved that the order-isomorphism in Shelah's theorem can be taken to be the restriction to $\mathbb{R}$ of an entire function. Using an approximation theorem of L. Hoischen, we also showed that the order-isomorphism f can be taken so that it and its first n derivatives approximate those of a given nondecreasing surjection g of class C^n. Hoischen's theorem also gives equality of the derivatives of f and g on a closed discrete set. We incorporate that improvement into our earlier result.

The following special case of the theorem is provable in ZFC. Let $g\colon \mathbb{R} \to \mathbb{R}$ be a nondecreasing C^n surjection. Let $\varepsilon\colon \mathbb{R} \to \mathbb{R}$ be positive and continuous. Let $E \subseteq \mathbb{R}$ be a closed discrete set on which g is strictly increasing. Let each of $\{A_i\}$, $\{B_i\}$ be a sequence of pairwise disjoint countable dense subsets of $\mathbb{R}$ such that for each $i \in \mathbb{N}$ and $x \in E$ we have $x \in A_i$ if and only if $g(x) \in B_i$. Then there is an entire function $f\colon \mathbb{C} \to \mathbb{C}$ such that $f[\mathbb{R}] \subseteq \mathbb{R}$ and the following properties hold.

(a) For all $x \in \mathbb{R} \setminus E$, $Df(x) > 0$.

(b) For $k = 0, \dots, n$ and all $x \in \mathbb{R}$, $|D^k f(x) - D^k g(x)| < \varepsilon(x)$.

(c) For $k = 0, \dots, n$ and all $x \in E$, $D^k f(x) = D^k g(x)$.

(d) For each $i \in \mathbb{N}$, $f[A_i] = B_i$.

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The commutant of $B(H)$ in its ultrapower

For an infinite-dimensional separable Hilbert space H let $B(H)$ be its algebra of bounded linear operators. Fix a free ultrafilter U on N and consider the ultrapower of $B(H)$. Eberhard Kirchberg asked whether the commutant of $B(H)$ in its ultrapower is equal to the scalar multiples of the identity. I will answer 3/4 of his question.

This is a joint work with N. Christopher Phillips.

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The consistency of $\mathfrak{b} = \kappa < \mathfrak{s} = \kappa^+$

Using a proper forcing notion, in 1984 S. Shelah obtained the consistency of $\mathfrak{b} = \omega_1 < \mathfrak{s} = \omega_2$. We obtain a σ-centered suborder of Shelah's poset which preserves the unboundedness of a given unbounded family and adds a real not split by the ground model reals. Thus under an appropriate finite support iteration of length κ^+ we obtain the consistency of $\mathfrak{b} = \kappa < \mathfrak{s} = \kappa^+$ for κ an arbitrary regular uncountable cardinal.

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Distinguishing Number of Countable Homogeneous Relational Structures

The distinguishing number of a graph G is the smallest positive integer r such that G has a labeling of its vertices with r labels for which there is no non-trivial automorphism of G preserving these labels.

M. Albertson and K. Collins computed the distinguishing number for various finite graphs, and W. Imrich, S. Klavzar and V. Trofimov computed the distinguishing number of some infinite graphs, showing in particular that the Random Graph has distinguishing number 2.

We compute the distinguishing number of various other finite and countable homogeneous structures, including undirected and directed graphs, and posets. We show that this number is in most cases two or infinite, and besides a few exceptions conjecture that this is so for all primitive homogeneous countable structures.

Joint work with L. Nguyen Van The and N. Sauer.

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Analytic sets in any dimension

We study the extension to infinite products of the following two dichotomies concerning analytic subsets of two Polish spaces: the Kechris–Solecki–Todorcevic dichotomy about graphs, and the Debs–Lecomte dichotomy about potentially $\mathbf{\Pi}^0_\xi$ sets.

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Weak Saturation of Ideals on $P_\kappa(\lambda)$

Suppose κ is an uncountable successor cardinal, $\lambda > \kappa$ is a cardinal of cofinality less than κ, and J is a κ-complete, fine, proper ideal on $P_\kappa(\lambda)$. Then, as shown by Chris Johnson and Yo Matsubara, $P_\kappa(\lambda)$ can be partitioned into λ many pieces not in J. What about getting more pieces, say $\mathrm{cof}\big(P_\kappa(\lambda), \subseteq\big)$ many? We use pcf theory to show that this can be achieved in a number of cases.

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Actions isométriques affines de groupes polonais sur les espaces de Banach sans points fixes

Cet ouvrage conjoint avec Lionel Nguyen Van Thé (Université de Calgary) propose une caractérisation des groupes topologiques possédant la propriété de point fixe lorsqu'ils agissent par actions isométriques affines sur les espaces de Banach. Entre autres, il est démontré que seuls les groupes précompacts possèdent cette propriété, et que dans le cas séparable, il suffit de considérer les actions sur un unique espace de Banach $\langle \mathbb{U} \rangle$, appelé l'espace de Holmes (l'unique espace de Banach engendré par l'espace métrique universel d'Urysohn $\mathbb{U}$). Enfin, nous montrons les exemples de groupes polonais qui ne peuvent pas agir de façon propre par isométries sur les espaces de Banach, voire les espaces métriques complets. Tels sont le groupe unitaire $U(\ell^2)$, le groupe symétrique infini S_∞, le groupe des homéomorphismes $\mathrm{Homeo}_+[0,1]$, et bien d'autres.

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